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for basic physics

Preliminaries: Things you have to know

Here are some preliminaries for this course. These are things that should be automatic to you. They don't really have anything to do with physics, and aren't necessarily something you'll learn in physics, but should already know from your preparation to be entering this technical class.

Basic Trigonometry. Know what a right triangle is and how sin, cos, and tan work with that right triangle. Know how the pythagorean theorem works with a right triangle.

Basic Trigonometry. Know that $\sin 0 = 0^\circ$, $\cos 0^\circ = 1$, $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$.

Basic Trigonometry. Know the difference between a radian and degree and how to interconvert between them.

Basic Calculus. Given a function $y(x)$, know how to find dy/dx and d^2y/dx^2. As an alternative notation, given $y(x)$, know how to find $y'(x)$ and $y''(x)$.

Vectors or Arrows. In physics we often draw arrows on objects to indicate that something is happening to it. The arrow is also called a "vector" and it'll be labeled with some quantity, like F for force or v for velocity, etc. For example if you see a ball with an arrow pointing up and to the right, and the arrow is labeled v, you might be able to conclude that the ball is moving in that direction.

Discussing Vectors. Vectors often need to be described in words. Know what it means for a vector to point at "30° with the $+x$-axis," "16° north of east," or "south east."

Operation on Vectors (1). If you have a vector, no matter what direction it is pointing, you should be able to find its x and y components. This is most easily done by drawing a small xy-coordinate system at the tail of the arrow. Next, treat the vector itself like the hypotenuse of a right triangle and draw in the legs, one along the x axis and the other along the y axis of your little coordinate system. Label in some angle and use sine and cosine as needed to find the lengths of the x and y components (the legs).

Operation on Vectors(2). If you have the x and y components of a vector, you should be able to draw the vector itself and determine the angle the vector makes with respect to the x-axis. This is all done with basic trigonometry. Invariably this will involve using $\tan^{-1}$ somewhere. You should also know how to find the magnitude of a vector, which comes from the Pythagorean Theorm; if you know the "legs" of a right triangle (the components), you should be able to find the hypotenuse (or the magnitude of the vector).

Handling Vectors. There are many ways of representing vectors; here are the most common.

- Magnitude and angle. Specify the magnitude (strength, length, etc.) and the angle. Like 10 m/s at 45° up from the $+x$-axis.
- $\hat{\imath}, \hat{\jmath}, \hat{k}$-notation (or engineering notation). Specify the components of the vector directly. $\hat{\imath}$ stands for x, $\hat{\jmath}$ stands for y and $\hat{k}$ stands for z. In this class the z-component will always be zero. So a vector written like $5\hat{\imath} + 2\hat{\jmath}$ means a vector that has a strength of 5 units in the x-direction and 2 units in the y-direction. You should be able to find the magnitude and angle of this vector (if needed) directly from the 5 and the 2.

- Ordered set notation. $< x, y, z >$ where x, y, and z are the components of the vector, so $< 5, 2, 0 >$ would be the same as the vector above.

Variables. Variables are letters that stand for numerical values. Something like x^2 means that is x is known, we should multiply it by itself. On paper, we could write $x = 4$, then know that x^2 will evaluate to 16. *Computers, calculators, and spreadsheets* behave in the same manner; textual variables can hold values for later use. So on a computer we could type `x=5` assigning the value of 5 to variable `x`. Variable names on computer are often longer, to make them more descriptive. So instead of `x` we might see `sphere_x`, meaning the x-coordinate of the sphere. In a spreadsheet (like Excel) the variable name might be something like `A9`, representing the cell at column `A`, row `9`.

Functions in mathematics. In mathematics, you should be familiar with the use of a function. For example if you know that $f(x) = x^2$, then you are free to *use* the function. You can differentiate it: $f'(x) = 2x$. You can evaluate it at $x = 5$, or $f(5)$ to get 25. Functions can also have different names, like g, and be functions of more than one variable, like $g(x, y, t) = x^2 + y^2 - t^2$ for example.

Functions on your graphing calculator or a computer. *With computer, calculators, or spreadsheets* there are also functions, but instead of just returning a number, like `sin(x)` a function on a computer can cause something to happen, like to color the screen or draw a vector. Function names on a computer are typically longer than just f or g, such as `draw_vector`. Computer functions often have parameters too, in the same format as their mathematical counterparts, as in *name* then *parenthesized list of parameters*. So instead of $f(x, y)$, we'd have *draw_vector(tip,tail,color,label)* where *tip* and *tail* are the tip and tail coordinates of a vector to draw, with a color of *color* and a label of *label*. Calling this function doesn't return a number; it draws a vector on the screen.

Vectors: More than "magnitude and direction"

When asked, most students will say that a vector is a "quantity with a magnitude and direction." There is much more to vectors than this textbook meaning, and the sooner you "become friends" with vectors, the easier time you're going to have in your core math and science classes.

To start, think of a vector as a "container" for information about an object. To emphasize the container aspect, we'll write vectors enclosed in a $<$ and $>$, or the "ordered set" notation (see above). As an example, an object might be located at $x = 5$, $y = 3$, and $z = -1$. In vector form, this would be written as $\vec{r} =< 5, 3, -1 >$, or $\overrightarrow{pos} =< 5, 3, -1 >$. In both cases, the 5, 3, and -1 are called the components (or parts) of the vector. The arrow over the symbol means it's a vector (it has the three components). Notice how compact the vector is as a container. One look at $< 5, 3, -1 >$ and you can immediately see the x, y, and z position of an object.

A velocity vector can be stated in the same manner. Suppose an object has an x-velocity of 6 m/s and a y-velocity of 2 m/s. It's v-vector could be written as $\vec{v} =< 6, 2, 0 >$, or $\overrightarrow{speed} =< 6, 2, 0 >$. Acceleration vectors can be written similarly. For example, an object in free fall has $\vec{a} =< 0, -9.8, 0 >$.

The real convenience of vectors is in their algebraic operations. For example, a vector can be multiplied by a scalar, by simply multiplying all of its components by the scalar. So, $5\times < 6, 2, 0 >=< 30, 10, 0 >$. This is useful in the physics equations $v = v_0 + a\Delta t$ and $x = x_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$, Because Δt is always a scalar (a time interval), but in these equations it's multiplied by either a or v_0, which can be vectors. This means that an a-vector of $< 0, -9.8, 0 >$ times a Δt of 2 seconds would be $< 0, -9.8, 0 > \times 2$ or $< 0, -19.6, 0 >$. But since $v = a\Delta t$, the $< 0, -19.6, 0 >$ is the object's new v-vector.

For a complete example, suppose $\vec{v_0} =< 2, 1, 0 >$ and $\vec{a} =< 0, -9.8, 0 >$. If you wish to know the object's new velocity after 2 seconds has gone by, you can use $\vec{v} = \vec{v_0} + \vec{a}\Delta t$, or $\vec{v} =< 2, 1, 0 > + < 0, -9.8, 0 > \times 2$. Working this, we'll get $\vec{v} =< 2, 1, 0 > + < 0, -19.6, 0 >$ or $\vec{v} =< 2, -18.6, 0 >$. In other words, after 2 seconds, the object is moving 2 m/s along the x-axis, 18.6 m/s along the $-y$-axis, and it is not moving at all along the z-axis.

The Common Stuff: your "magnitude and direction"

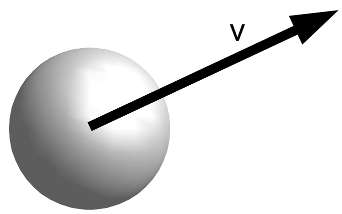
The notation $< x, y, z >$ is a very succinct vector notation.All of the common vector stuff can be found by simply using the information in such a vector. Two important "stuffs" are magnitude $\sqrt{x^2 + y^2 + z^2}$ and angle in the xy-plane, $\tan^{-1}(y/x)$. For example, suppose $\vec{v} =< 3, 2, -1 >$. The magnitude of v can be found by $|v| = \sqrt{3^2 + 2^2 + (-1)^2}$, or $|v| = 3.7$. The angle the v vector is oriented at in the xy-plane is found from $\theta = \tan^{-1}(2/3)$ or $33.6°$ with respect to the $+x$-axis.

The velocity-vector: The theme of this entire class

Look at this figure, an object with a velocity vector pointing from it. There is a twofold theme for this entire class that comes from this figure.

1. The v-vector tells us the direction the object is currently moving and about where it will be a small interval of time in the future.

2. This entire class is about different laws of physics that allow us to *make the v-vector of an object change, either in direction or length, or both.*



- Why is understanding how a v-vector is manipulated so important? Because basic physics is about understanding how objects move. A key element in this understanding is in being able to *predict* where an object will be at a given time in the future, and to be correct in your prediction. All of this is contained in an object's v-vector.

- Changing the *magnitude (or length)* of the v-vector changes the speed of an object. Making the length grow means the object is moving faster. Shrinking the length means the object is moving slower. Changing the *direction* of a v-vector changes the direction in which the object will move over a small time interval into the future. The v-vector dictates the impending motion of an object. Manipulating it, or understanding it is the key to controlling or understanding an object's motion.

- To really understand each lesson this quarter ask yourself: "Do I understand how the physics in this lesson can change the v-vector of an object?"

Week 1: One-dimensional Motion

The v-vector of an object will be changed by: Applying an acceleration either parallel or anti-parallel to v. The goal of this week is to understand how acceleration, a, can be used to primarily change an object's v-vector, and secondarily force changes in an object's position (x), all in just one-dimension. An "object" means anything that can move, like a ball, car, truck, or person. One-dimensional means the object will only move along a straight line, typically along the x-axis if it's moving left or right or y-axis if it's moving up or down. There are two equations you need for this, $x = x_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$, and the second is $v = v_0 + a\Delta t$. If the object is at x_0 with speed v_0, then x and v will be the object's new position and speed at some time interval Δt later. These two equations allow you to compute the new position and speed of an object (x and v), based on its old position and speed (x_0 and v_0), given some acceleration a that is acting on the object, over a time interval Δt. Δt is sometimes called the "time step" and is a small interval of time that separates when the object has x_0 and v_0, and when it will have x and v. We said that a drives changes in v and x. Notice in these equations if $a = 0$, then $v = v_0$, meaning that v doesn't change between time steps; v is constant if $a = 0$. In order for v to change, a must be nonzero. In other words, an object's speed can change only if it has an acceleration. For the x-axis (left-right motion), we have that $x = x_0 + v_{0x}\Delta t + \frac{1}{2}a_x\Delta t^2$ and $v_x = v_{0x} + a_x\Delta t$. For the y-axis (up-down motion), we have that $y = y_0 + v_{0y}\Delta t + \frac{1}{2}a_y\Delta t^2$ and $v_y = v_{0y} + a_y\Delta t$. These equations are the same, just the notation is different, being very specific as to the axis which axis it pertains. Suppose you have a sphere at $x = 5$ m with speed $v = 1$ m/s and an acceleration of $a = 0.5$ m/s^2. When the next frame comes up, say $\Delta t = 0.1$ s later, where will the sphere be and what will its speed be? Use the equations to get that $x = 5\text{m} + (1\text{m/s})(0.1\text{s}) + (0.5)(0.5\text{m/s}^2)(0.1\text{s})^2$ or $x = 5.1025$ m and $v = 1\text{m/s} + (0.5\text{m/s}^2)(0.1\text{s})$ or $v = 1.05$ m/s. You can iteratively use this new x and v as a new x_0 and v_0 (i.e. $x \to x_0$ and $v \to v_0$) for computing still another x and v another Δt in the future. Can you find x and v after another Δt has gone by (ans: $x = 5.208$ m and $v = 1.06$ m)? Be very aware of signs. Think of a cartesian coordinate system with $+x$ to the right, $-x$ to the left, $+y$ up and $-y$ down (assume Δt is always positive). Positive values of position mean the object is to the right (x) (or up, y) relative to the origin. Negative means the object is left (x) (or down, y) relative to the origin. Positive values of speed mean the object is moving toward the right (v_x) or up (v_y), negative means to the left (v_x) or down (v_y). The sign of a alone doesn't immediately help to characterize the object's motion. If, however, a and v have the same sign, $v = v_0 + a\Delta t$ will predict an increase in v (that is if v and a have the same sign, an object will speed up). Likewise, an object will slow down if v and a have opposite signs. A case where opposite signs of v and a persist means v will get smaller and smaller, until eventually $v = 0$ at which case the object will stop. If a still persists, then v will begin to increase in the same direction as a; now the object is speeding up, but in the opposite direction to its original motion. All told the object slowed down, stopped, then started speeding up in the opposite direction. All combinations of signs between v and a are possible. $v > 0$ and $a < 0$ is a slow-down and potential turn-around case, as is $v < 0$ and $a > 0$. $v > 0$ and $a > 0$ or $v < 0$ and $a < 0$ are speed up cases, but in opposite directions. Lastly, you should be able to draw arrows on an object, representing its v and a and that instant. An arrow should point in the direction of the parameter it represents, and its length should be proportional to its amount (or strength). For example, if on an object the arrow for v and the arrow for a were opposite, you'd know the object was slowing down. An object going 2 m/s would have a v arrow half as long as one going 4 m/s. **Book reference: Sections 2.4 and 2.5.**

Week 2: Two-dimensional Motion

The v-vector of an object will be changed by: Applying a downward, vertical acceleration to the object. The goal of this week is to understand how objects move in fully two dimensions. Last week you concentrated on motion strictly along the x or y axis. Two dimensional motion is where an object undergoes motion along the x and y axes *at the same time*. The position of an object in two-dimensional space can be plotted by its (x, y) coordinate. These coordinates are found by the equations $x = x_0 + v_{0x}\Delta t + \frac{1}{2}a_x\Delta t^2$ and $y = y_0 + v_{0y}\Delta t + \frac{1}{2}a_y\Delta t^2$. Note that also evolving as an object moves are its speeds along two axes as well, $v_x = v_{0x} + a_x\Delta t$ and $v_y = v_{0y} + a_y\Delta t$. Remember that the x and y coordinates are perpendicular to each other, that is the x and y axes are orthogonal. This is a special relationship in math and physics, and means that processes along one axis do not affect processes along the other axis. Therefore, whatever happens along the x axis does not affect what happens along the y axis, and vice-versa. This is a key concept to understand this week. Two-dimensional motion is sometimes called "projectile motion" which encompasses objects flying through space under the influence of gravity. Baseballs, cannon balls, basketballs moving through space are all examples of projectile motion. The movies this week will show projectiles in flight, restricted to motion where $a_x = 0$ and $a_y = -g = -9.8$ m/s^2. You can immediately find forms of the $x =$ and $y =$ equations above, given these restrictions. An any given time, your object will have four quantities describing its motion: x, y, v_x, and v_y. Since position and speed now each have two components (or parts), position and speed will be "vectors," called $\vec{r}$ and $\vec{v}$ respectively. $\vec{r}$ will consist of two components, the x and y coordinates of the object. Similarly, $\vec{v}$ will consist of the components v_x and v_y. As you will now see, the two components of both $\vec{r}$ and $\vec{v}$ gives them both a magnitude (strength, length, etc.) and direction, which you must know how to handle. There are two ways of dealing with vectors, and you should be proficient with both. The first way is in "magnitude-angle form," where you report the magnitude of the vector and the angle at which it is pointing. For the position, the magnitude (or total distance from the origin) is $r = \sqrt{x^2 + y^2}$. The angle this vector will make relative to the $+x$-axis is given by θ where $\theta = \tan^{-1}\frac{|y|}{|x|}$. The absolute value signs are important to remove any negative values that might pop up and ensure the angle is with respect to the $+x$-axis. The velocity vector is tracked similarly, namely $v = \sqrt{v_x^2 + v_y^2}$ with $\alpha = \tan^{-1}\frac{|v_y|}{|v_x|}$, where α is the angle the velocity vector makes with respect to the $+x$-axis, and is essentially the direction the object is moving in at that instant of time. Be sure you understand why a vector has a magnitude and an angle, and be sure you can always compute both from a given vector's components. The other way of handling a vector is in "component form." In this form, you list each component directly, next to a unit vector specifying what axis the component goes to. So if an object is 5 meters along the x-axis and 2 m along the y-axis then $\vec{r} = 5\hat{\imath} + 2\hat{\jmath}$, where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors meaning x-axis and y-axis, respectively. The other type of two-dimensional motion that is important is circular motion, which describes how an object moves in a circle. In this type of motion, the object is always has an acceleration that points toward the center of the circle around which it traveling. If you choose a circle of radius r and want the object to move around the circle with a speed v, then the strength of the acceleration, called the "centripetal acceleration" must be $a = \frac{v^2}{r}$, and it must always point toward the center of the circle. **Book reference: Sections 3.3 and 3.4**

Week 3: The Basics of Newton's Laws

The v-vector of an object will be changed by: Applying a force or a net-force on an object. The goal of this week to use Newton's Laws to see that accelerations actually come from forces applied to an object. In the past two weeks, a was simple a "given" quantity. It simply existed in the equations of motion and you were allowed to give it any value. This week we will see that accelerations come from *forces*. Forces are pushes or pulls on objects that you witness everyday (push a door to open it, pull on your book to lift it, push a cell phone button to click it). With the exception of gravity, forces are always "contact forces" meaning a force must actually touch an object to exert its influence on it. Forces also require an agent, meaning that you should always be able to identify what (the agent) is producing the force. Forces are also vectors, meaning their strength (push or pull) can be in any direction. You probably know that Newton's Law says $F = ma$, but this is a horrible equation to ever try and use in a physics course. $a = \frac{F}{m}$ is better, but still isn't quite right. It is more correct to say that $a = \frac{\Sigma F}{m}$, which still isn't fully correct. The best version is $\vec{a} = \frac{\Sigma \vec{F}}{m}$, stressing the vector property of forces. Be sure you fully understand what this last version means and how to use it. $\vec{a}$ is the acceleration, m is the object's mass and $\Sigma \vec{F}$ is the sum of all forces acting on the object. Mass is the amount of "stuff" an object is made from and never changes unless portions of the object are somehow broken off. We will only be concerned with five forces: weight, tension, normal, friction, and drag. Weight is $w = mg$, where m is an object's mass and g is the earth's acceleration of gravity of 9.8 m/s^2. Do not confuse mass and weight; they are not the same thing and be sure you know the difference between them. Mass is also known as object's inertia, or resistance to want to change its current state of motion. It would hurt if you placed a bowling ball on the floor in front of you and kicked it as hard as you can. Would it also hurt to kick the bowling ball in the middle of outer space where $g = 0$? Tension is the tug an object feels when pulled by a rope attached to it. Normal is the force an object feels when it is sitting on a surface, is always perpendicular to the surface, and is not always equal to mg. Friction is a force that always opposes all motion; it always acts in a direction exactly opposite to that in which an object is moving (or trying to move), and typically comes when the object rubs or drags against another object as it moves. It is defined as $\vec{f} = \mu \vec{N}$ where μ is the coefficient of friction (p. 150 in your book), and $\vec{N}$ is the normal force acting on the object. Drag is like friction in that it always acts in a direction opposite to that in which an object is moving, but comes from air or water, through which an object might be moving. Drag, $D = Cv^2$, where C depends on the shape and size of the object, and v is the object's speed. Air resistance is a type of drag force. The crux of this entire week is the part, $\Sigma \vec{F}$, because it requires three hard steps. The first, which most students have great difficult with, is to identify all forces acting on an object. The second is to correctly draw these forces, each pointing in the proper direction, as they act on the object (even more difficult for most students). The third is to realize that $\Sigma \vec{F}$ which is only useable when you break it up into component form, or ΣF_x and ΣF_y. Your working equations for this week are then $a_x = \Sigma F_x/m$ and $a_y = \Sigma F_y/m$. The connection points with weeks 1 and 2 then are that these accelerations, which come from forces, are the same a's that go into the equations of motion for x and y. Thus, $x = x_0 + v_0\Delta t + \Sigma F_x \Delta t^2/(2m)$ and $v_x = v_{0x} + \Sigma F_x \Delta t/m$. Be sure these make sense to you and do not causally read over the ΣF_x and ΣF_y. Know what they mean: Using Newton's law really means that all forces acting on an object need to be broken up into their x and y components, properly signed (+ or $-$), then added together along a given axis. **Book reference: Sections 4.1, 4.3, 5.2, 5.3**.

Week 4: Simple machines that obey Newton's Laws

The v-vector of an object will be changed by: Identifying the net-force on an object, that is likely linked to another object. Objects connected by ropes and pulleys. Ropes in freshman physics are always massless and these rules apply. 1) Ropes always pull away from their points of contact. That is, tensions in ropes are always drawn pointing away from the point where an the rope connects to an object, along the rope itself. 2) You cannot push on a rope. Ropes may only be pulled on. 3) Objects connected by ropes will always have the same *magnitude* of v (velocity) and a (acceleration), although the algebraic signs of v and a might be different for each object. 4) The tension in a rope connecting two objects is the same throughout the rope. 5) Tensions on opposite ends of a rope must have opposite algebraic signs for use in any equations. That is, the tensions on either ends of a rope that are pulling on their respective objects, always point toward each other, along the rope. This is the only way in which both ends can *pull* on the objects to which they are connected. 6) If a rope passes over a massless pulley, the magnitude of its tension will not change. A massless pulley just changes the direction of the rope, hence tension. That is tension on one side of a pulley will be the same on the other side, save for the opposite algebraic signs required. 7) If a rope passes over a real pulley (with a defined mass and radius), then the tension on one side of the pulley *does not* in general equal to the tension on the other side of the pulley, and you should not assume it is. The opposite sign rule applies to the different tensions. **Objects on sloped surfaces.** If an object is on a surface sloped at an angle θ, there will be a downward "sliding force" on the object of magnitude $mg\sin\theta$, in a direction pointing down and parallel to the slope. This sliding force is what causes objects on sloped surface to want to slide or roll downhill. Gravity g is the originator of this force. **Objects that interact with springs.** Suppose a spring has a free end and a fixed end. The free end can "spring along" the direction of the spring itself, and the fixed end cannot move at all. Suppose also when nothing is touching the spring (when the spring is in equilibrium), the free end is physically located at position s_0. When the free end of the spring is displaced to a position s, the force the spring exerts on an object connected to its free end is $F=-k(s-s_0)$. This is Hooke's Law. The minus sign indicates that the spring force always opposes the direction of s relative to s_0, and k is the "spring constant" or the stiffness of the spring (the larger k the stiffer the spring). **Weight.** When an object that has a mass m is in a gravitational field, it will have a force on it called weight, which is $W=mg$. This force always points straight down, no matter what other orientation or situation the with which the object might be involved. **Normal force.** When an object with weight W is placed on a surface, it disrupts the equilibrium position of the molecular structure forming the surface on which the object is placed. The desire of these molecules to want to return to equilibrium causes them to push back on the object with a force called the normal force. The normal force is *always* perpendicular to the surface on which the object is sitting. It is tempting to call the magnitude of the normal force mg, or the weight of the object, but this only true when the object is sitting on a flat surface. In other situations, the magnitude of the normal force can only be found by summing forces perpendicular to the surface and setting the sum equal to zero, then solving for N. **Friction.** Kinetic friction is a force that is always oriented exactly opposite to an object's v and has a magnitude of $f=\mu N$, where μ is the coefficient of kinetic friction and N is the normal force on the object, due to the surface on which it sits. **Centripetal force.** If a situation arises where a force F is locked at 90° with respect to an object's v-vector, then the force is called a centripetal force. The force will cause the object to move in a circle of radius R if the force magnitude is mv^2/R, where m is the mass of the object.

Week 5: Work, Kinetic Energy, Potential Energy, and Conservation of Energy

The v-vector of an object will be changed by: Adding or removing kinetic energy from an object. No one know what energy is, but it can be compared to money and time (you can lose, gain, save, waste, or spend them, etc.), and we all "know" what energy, money, and time are until someone asks us to tell them what they are! We'll focus on two types of energy, Kinetic energy (KE) and Potential energy (PE), a way of "processing" energy, called "work," (W) and a guiding principle, called "conservation of energy." The units of energy (KE, PE, and W) are in Joules, or J. KE is energy something (of mass m) has because it it moving with some speed v, or $KE = \frac{1}{2}mv^2$. If an object is moving it has KE; if it is at rest, it doesn't. PE is stored energy that has not been released yet to do something. Our society is usually concerned with chemical or nuclear PE (oil/gasoline, natural gas, nuclear power plants), but in this class we'll only concern ourselves with mechanical PE, and further, only three types of it. Gravitational PE, or $U_g = mgh$, spring PE, or $U_{sp} = \frac{1}{2}k(x - x_n)^2$, and pendulum PE, $U_{pend} = mg(1 - \cos\theta)$. U_g is PE an object has because it is not trapped at some lowest possible position to which it may fall. This lowest PE position can be tricky to identify and is not always at ground level. You must examine an object's position and ask yourself: "If the object was carefully placed at rest, at this position, would be able to fall down any farther if given a small nudge?" In U_g, mg is the weight of the object and h is its vertical distance above the lowest possible position. U_{sp} is energy a compressed or expanded spring may store, where k is the spring constant, x is how far the end of the spring has been expanded or compressed past its "natural" position at x_n. If $x = x_n$, then the spring is neither expanded or compressed, and $U_{sp} = 0$. If $x > x_n$ or $x < x_n$ then $U_{sp} \neq 0$. Pendulums are any mass that can swing from a very light rope attached to some higher point and are great examples of objects whose PE is zero when the mass is not on the ground. In the equation for U_{pend} above, θ is the angle the pendulum makes with respect to the vertical (where the mass is directly below the upper attachment point of the rope, when $\theta = 0$). Work is a way of using a force to inject or remove energy to or from an object. For us, $W = F \cdot \Delta r \cos\theta$, where F is a force applied to the object, Δr is how far the object moved while the force was applied, and θ is the angle between the force and the direction of Δr. if $W > 0$ then energy will be added to the object, if $W < 0$ then energy will be taken from the object. For us, F and Δr will always positive; the sign of $\cos\theta$ will determine the sign of W. Friction always results in energy loss or $W < 0$. Lastly, we have the law of "conservation of energy" (CE). CE states that the sum of KE and PE is always a constant. This means if PE goes up, KE must go down and if PE goes down, KE must go up, both in such a way to keep $KE + PE =$ a constant. The "constant" is the total energy of the system. The most useful form of this law is that in realizing that if the sum of KE and PE are constant, then the sum of KE and PE, say at some point A in the object's motion will be the same as the sum of KE and PE at some point B in the object's motion, or $KE_A + PE_A = KE_B + PE_B$. Also, since KE is always $\frac{1}{2}mv^2$, then $\frac{1}{2}mv_A^2 + PE_A = \frac{1}{2}mv_B^2 + PE_B$, which, if you just fill in your associated PE function begins to form a useful "physics equation" that can be used to solve problems. Work ties into this all by showing where energy is injected or is lost by the object. Here's a useful form that includes work: $\frac{1}{2}mv_A^2 + PE_A + W = \frac{1}{2}mv_B^2 + PE_B$, which shows how if $W > 0$ (energy into the object) will lead to a greater total energy represented on the left side of the equation. $W < 0$ would lead to a smaller total energy on the left hand side. **Book reference: Sections 6.1, 6.2, 7.1, 7.2**.

Week 6: Momentum and Conservation of Momentum

The v-vector of an object will be changed by: Causing an object to interact (or collide) with another object. In all of the previous weeks, we concerned ourselves only with isolated objects. This week, we'll see what happens when two (or more) objects interact which each other, in the form of contact between the two bodies (as in a collision, or in the sudden motion of two or more objects due to a need for them to separate due to an explosion). When two bodies come in contact with each other, each exerts a pushing force on the other (think of the last time you bumped into someone in a crowded place: you felt a push, and so did they). **Further, the force that one exerts on the other is always exactly the same.** This is Newton's third law of "equal and opposite reaction forces." For example, if two cars collide, during the collision, car A will exert a force on car B (F_{AB}), and car B will exert the exact same force on car A (F_{BA}). The two forces will have the same strength, but be in exactly opposite directions to one another. In other words, $F_{AB} = -F_{BA}$. It doesn't matter is one car is heavier (more massive) than the other. The push force from one car will equal the push force from the other. What if one car is a small Honda and the other car a huge SUV? If so, when in contact, the force the Honda exerts on the SUV will be equal to the force the SUV exerts on the Honda, only in the opposite direction. What about a bug hitting a car windshield? The force of the bug on the windshield is equal to the force of the windshield on the bug, only in the opposite direction. Why then does the bug get crushed and the SUV doesn't even feel the collision? Because the resulting motion *after the collision* is driven by the acceleration the body takes from the collision, while in contact with the other object. Suppose the equal and opposite force of the bug-windshield collision is 0.1 N. The bug has a mass of 0.001 gram, or 1×10^{-6} kg. It's resulting acceleration will be $a = F/m = 0.1$ N$/1 \times 10^{-6}$ kg $= 100,000$ m/s^2. The SUV, with a mass of about $4,000$ kg gets an acceleration of $a = 0.1$ N/4000 kg $= 0.000025$ m/s^2. Collisions are typically very brief, say 1 ms, or 0.001 s. During this time, a parameter called "impulse" exists, defined as $J = \Delta p$, which is the change in an object's momentum. How far does each move in this time? The bug will move 5 cm, the SUV will "move" about the diameter of of an atom making up the windshield. The bug get crushed because its internal structure cannot sustain an acceleration of about $10,000$g. This equal and opposite force idea leads to momentum, which is defined as $p = mv$ or more correctly, $\vec{p} = m\vec{v}$. Notice $\vec{p}$ involves velocity directly. We also have "conservation of momentum" that says that $\vec{p}_{before} = \vec{p}_{after}$. The "before" and "after" refer to before and after a collision. This law itself allows us to ignore the physics *of the collision* and instead focus on the physics *just before and just after the collision.* More correctly, the law is $\Sigma\vec{p}_{before} = \Sigma\vec{p}_{after}$, indicating that the law means add all objects carrying momentum before a collision and set equal to the sum of all momenta carrying objects after the collision. Since p is a vector, so you must sum the momenta of all objects in the x direction, then in the y direction, both before and after the collision in order the the conservation law to be helpful. Lastly, there are two types of collisions, elastic and inelastic. In elastic collisions, the colliding objects bounce off of each other, while in inelastic, they all stick together creating a new "conglomerate mass" which is the sum of the individual masses that stuck together. In applying the conservation law for an inelastic collision, you typically have something like $m_1\vec{v}_{1before} + m_2\vec{v}_{2before} + m_3\vec{v}_{3before} + ... = (m_1 + m_2 + m_3 + ...)\vec{v}_{after}$. Notice that there's only one velocity after the collision ($\vec{v}_{after}$) because only the "big blob" is moving after they all collided and stuck together. For an elastic collisions, where two objects (1 and 2) collide along a single axis, we'll have $v_{1after} = \frac{m_1-m_2}{m_1+m_2}v_{1before} + \frac{2m_2}{m_1+m_2}v_{2before}$ and $v_{2after} = \frac{2m_1}{m_1+m_2}v_{1before} + \frac{m_2-m_1}{m_1+m_2}v_{2before}$. **Book reference: 8.2, 8.3, 8.4**.

Week 7, Part 1 of 2: Observing Rotational Motion

The v-vector will now become an ω vector. ω will be changed by: Applying an angular acceleration either parallel or anti-parallel to ω. Thus far, we've discussed objects moving in straight lines, or "linear motion." Now we'll discuss "rotational motion," or how an object rotates or spins. Think of a merry-go-round, rolling wheel, or a pulley that actually turns as it guides a rope. For the rotating object, you should always be able to identify the axis about which it rotates, called the "axis of rotation," which might be through its center, but not always. Given what you know by now about straight line motion (x, y, F, etc.), much of the core concepts here can be taught by analogy. Linear motion, has three working variables: x (or y), v, and a, with units of m, m/s, and m/s^2. In rotational motion we aren't concerned with how many meters an object has moved, but how many degrees (or radians) it has rotated through, so for angular position we'll have θ, angular speed ω, and angular acceleration, α. By analogy, $x \leftrightarrow \theta$, $v \leftrightarrow \omega$, and $a \leftrightarrow \alpha$, so instead of $x = x_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$, we'll have $\theta = \theta_0 + \omega_0\Delta t + \frac{1}{2}\alpha\Delta t^2$, and instead of $v = v_0 + a\Delta t$, we'll have $\omega = \omega_0 + \alpha\Delta t$. These are your core time-stepping equations for rotational motion. Here's an example (same numbers from week #1): A wheel has spun through $\theta = 5$ rad with an angular speed $\omega = 1$ rad/s and an acceleration of $\alpha = 0.5$ rad/s^2. How far will the wheel have spun $\Delta t = 0.1$ s later? Use the equations above to get that $\theta = 5\text{rad} + (1\text{rad/s})(0.1\text{s}) + (0.5)(0.5\text{rad/s}^2)(0.1\text{s})^2$ or $\theta = 5.1025$ rad and $\omega = 1\text{rad/s} + (0.5\text{rad/s}^2)(0.1\text{s})$ or $\omega = 1.05$ m/s. Like week #1's equations, you can compute a new θ and ω over the time step Δt, and can iteratively put $\theta \rightarrow \theta_0$ and $\omega \rightarrow \omega_0$ and use the equations again to compute the next θ and ω of the spinning object another Δt in the future. Also like week #1, be very aware of signs. θ can be positive or negative. Arbitrarily, we'll interpret a positive θ as a clockwise rotation and a negative θ as a counterclockwise rotation. With this sign convention, you can also place signs on ω and α. A positive ω means the object is rotating clockwise; a negative ω counterclockwise. If ω and α have the same sign, the object is spinning faster and faster. If ω and α have different signs, the object is spinning, but slower and slower. It may reach $\omega = 0$ in which case ω will start building up again in the same direction as α and acquire the same sign as α. The object will start rotating, faster and faster in the same direction as the original α. There is one last confusing point about the rotational world of θ, ω, α and the linear world of x, v, a. Think of the wheel on a car. A carbon atom (in the rubber) near the outer edge of the tire and one very close to the axle have the same ω, since they both rotate by the same amount in a given Δt. If they didn't the tire would warp and break apart. But the atom near the outer edge must have a linear speed (v) which is larger than the inner atom since it has a larger circle ($2\pi r$) to travel through on its way around. So although the atoms in the rubber have the same rotational speed ω, their linear speeds (v) are different. In fact, the v's scale with the distance from the axis of rotation, or, $v = r\omega$, where r is the distance from the axis of rotation. That is, if an atom is 5 cm from the axle, and the other is 10 cm from the axle, the latter has a v that is twice as large as the former. Similarly, $x = r\theta$ and $a = r\alpha$; linear distance and acceleration scale with r too. So you can describe a rotating object using the linear parameters x, v, a, but they aren't the most convenient, so we use θ, ω, and α. But each is related to the other via linear-variable = r(angular-variable), so they're really one in the same. To close, θ, ω, and α are the parameters that allow you to *observe* an object rotating. You'll see it rotate so far (θ) at some speed (ω). If the speed seems to be changing (speeding up or slowing down), then you can conclude that the object must have some α. **Book reference: 9.1, 9.2**.

Week 7, Part 2 of 2: What causes a rotation to occur?

The v-vector is now an ω vector. ω will be changed by: Applying a torque or net-torque to an object. α drives rotations, since if you have $\alpha \neq 0$, over successive Δt's, the α can lead to changes in ω, which together can lead to changes in θ. Here we address where α comes from. Just like $a = F/m$, here we'll have that $\alpha = \tau/I$, where τ is the torque on on object and I is the moment of inertia of the object. m is the mass of an object, or a measure of its resistance to want to change its state of motion, I is the resistance of an object to change its state of rotation (if it's not rotating, it wants to stay *not rotating*, etc.). Where mass is usually given for an object (so many kg's), I comes from simple formulas that resemble $I = cmR^2$, where m is the mass of the rotating object, R is the maximum extent of an object away from its axis of rotation, and c is a number like 1/2, 2/5, etc. Don't think of R as "radius;" an object doesn't have to be round in order to rotate. Look in your book for a chart of I's for objects rotating in various ways. Torque (τ) is like a "rotational force." From the discussion above, α drives rotational motion, because with α, a ω will develop, which will develop a θ. Since $\alpha = \tau/I$, you must have a τ in order to get an α. So where do torques come from? Forces. You must ultimately apply a force to an object to get it to rotate, but it matters 1) where you apply the force 2) at what angle you apply the force. This is all seen in the equation for torque, $\tau = rF \sin\phi$, where F is the force you apply to the object you wish to rotate, r is the length of a line that directly connects the axis of rotation and the spot where the force is applied. ϕ is the angle between the direction the force is applied and the axis-force connector line. This torque equation can be wholly understood by thinking of how one opens a door. If you push near the hinges the door won't open since $r \approx 0$, meaning $\tau \approx 0$ meaing $\alpha \approx 0$. If $\alpha \approx 0$ and the door is not already rotating then $\omega_0 = 0$ and $\omega = \omega_0 + \alpha\Delta t$ will never give any appreciable ω, no matter how long you wait (Δt), since $\alpha \approx 0$. Lastly, if $\omega \approx 0$ and $\alpha = 0$, then θ will always equal to θ_0, meaning the door will remain in the same rotational position. In other words "the door won't open." You can also push on the edge of the door, farthest from the hinges, maximizing r, but if you push directly on the narrow edge of the door (toward the hinges), $\phi = 180°$, once again, giving $\tau = 0$ (since $\sin 180° = 0$). This also gives $\alpha = 0$, like above, meaning that getting the door to swing (or allowing it to acquire some ω or $\theta \neq \theta_0$) will simply never happen. The best place to push on a door is farthest from the hinges, maximizing r, and perpendicular to the door, making $\phi = 90°$. This will give some non-zero value of τ, which will give a non-zero value for $\alpha(= \tau/I)$. With a non-zero α, an ω of the door will start to develop as θ will begin to become different that θ_0: the door will rotate. So certainly θ, ω and α track the observable rotation of an object, driven by α. But α must come from somewhere, and it comes from a torque, which ultimately comes from a force applied to an object (at some distance at some angle). I factors in to how hard it is to get the door to swing. A heavy, solid wooden door (front door of your house) is harder to open than a similarly sized light hollow door (on your bathroom) because m is larger and $I \sim m$. Thus for a given τ (your hand), α would be smaller since $\alpha = \tau/I$. Now say you had two doors that had the same mass, but one was two times wider than the other. Since $I \sim R^2$, where R is the maximum extent of the door, the door that is twice as wide would be four times harder to swing for a given torque applied, for the same reason. $\alpha = \tau/I$. The wider door, with the larger I, gives a smaller α. So α's, which drive all rotations, come from torques, just like a's, which drive all motion, come from forces. **Book reference: 9.4, 10.1**.